Bulk gravitons from a cosmological brane

David Langlois
GReCO, Institut d'Astrophysique de Paris, CNRS, 98bis Boulevard Arago, 75014 Paris, France

Lorenzo Sorbo
Laboratoire de Physique Théorique LAPTH, 9, Chemin de Bellevue, B.P. 110, 74941 Annecy-le-Vieux, France

(Received 27 June 2003; published 17 October 2003)

We investigate the emission of gravitons by a cosmological brane into an anti–de Sitter five-dimensional bulk spacetime. We focus on the distribution of gravitons in the bulk and the associated production of "dark radiation" in this process. In order to evaluate precisely the amount of dark radiation in the late low-energy regime, corresponding to standard cosmology, we study numerically the emission, propagation and bouncing off the brane of bulk gravitons.

DOI: 10.1103/PhysRevD.68.084006 PACS number(s): 04.50.+h, 98.80.Cq

I. INTRODUCTION

In recent years, a new cosmological scenario has emerged, based on the assumption that our universe is a *brane*: a subspace embedded in a larger, bulk space, with additional dimensions. In this context, a model that has attracted particular attention is that of a self-gravitating three-brane embedded in an empty five-dimensional spacetime [1], and especially its extension [2,3] inspired by the (noncosmological) Randall-Sundrum model [4].

In the latter case, where the bulk is endowed with a negative cosmological constant, it is possible to find a viable cosmological model, compatible so far with the available data. Because of the cosmological symmetries, it can be shown that in this model the most general bulk geometry corresponds to a portion of five-dimensional anti–de Sitter–Schwarzschild (AdS-Sch) spacetime, described by the metric

$$ds^2 = -\left(k+\mu^2 r^2 - \frac{\mathcal{C}}{r^2}\right)dt^2 + \left(k+\mu^2 r^2 - \frac{\mathcal{C}}{r^2}\right)^{-1} dr^2 + r^2 d\Sigma_k^2, \tag{1}$$

with $k=-1,0,1$ depending on the curvature of the three-dimensional spatial slices. The bulk cosmological constant is related to the mass scale μ via

$$\Lambda = -6\mu^2. \tag{2}$$

From the point of view of the brane, whose energy density is supposed to be the sum of an intrinsic tension σ and of the usual cosmological matter energy density ρ, cosmology is governed by the brane Friedmann equation, which is different from the standard one and reads [3]

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \left(\frac{\kappa^4}{36}\sigma^2 - \mu^2\right) + \frac{\kappa^4}{18}\sigma\rho + \frac{\kappa^4}{36}\rho^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}. \tag{3}$$

This result can be obtained by applying the gravitational junction conditions for a moving brane in Eq. (1), the cosmological scale factor $a(t)$ being simply the radial coordinate r at the brane position. The last term in Eq. (3), $\mathcal{C}/a^4$, is usually called "dark radiation" or "Weyl radiation" and represents the influence of the bulk geometry on the cosmology in the brane.

If the bulk is strictly AdS-Sch then $\mathcal{C}$ is a constant and corresponds to the five-dimensional mass in the Schwarzschild AdS metric (1). The term $\mathcal{C}/a^4$ then behaves exactly as a radiation component. The main constraint on the amount of such extra radiation comes from nucleosynthesis, since the primordial abundances of light elements depend crucially on the balance between the expansion rate of the universe and the rate of microphysical reactions. The good agreement of observations with the predictions of standard nucleosynthesis implies an upper bound on the number of nonstandard relativistic degrees of freedom in the universe and thus, in the context of brane cosmology, an upper bound on the value of the Weyl parameter $\mathcal{C}$.

Now, if the bulk is not strictly empty but contains a nonvanishing bulk energy-momentum tensor, then the Weyl parameter $\mathcal{C}$ is no longer necessarily constant. In other words, the generalization of Birkhoff's theorem to brane cosmology no longer applies. This has been illustrated in particular in models with a bulk scalar field [5,6].

In fact, in a realistic brane cosmological scenario, there is an unavoidable bulk component, which is simply due to the the gravitational waves, or bulk gravitons, produced by the brane matter fluctuations. Such bulk gravitons are created by the scattering of ordinary matter particles confined on the brane. The expected consequence of this phenomenon is to "feed" the Weyl parameter by means of a transfer of energy from the brane into the bulk.

This problem has been recently investigated in [7–9]. Whereas the authors of [9] analyze generic forms of transfer of energy between brane and bulk using a four dimensional effective description, the case of pure graviton emission has been studied in HM [7] and LSR [8].

In HM, the authors consider a pure AdS bulk and analyze the generation of dark radiation by considering two distinct phases. In the low energy regime ($\rho \ll \sigma$) the amount of generated dark radiation is obtained by directly equating it to the loss of energy density on the brane. In the high energy regime, the emitted gravitons are considered to remain gravitationally bound to the brane for the whole duration of this era, bouncing several times off it. To estimate the corresponding amount of dark radiation, the lost energy is reduced by a factor due to the energy dissipation resulting from the repetitive collisions with the brane. Because of the somehow crude distinction between the high and low energy eras, this analysis cannot follow the details of the transition between these two regimes. Moreover, the authors of HM give their final result up to an uncertainty factor, estimated to be between 0.5 and 1, due to the multiple bouncing of gravitons off the brane.

In LSR, a five-dimensional *exact* solution for the bulk is used, taking into account an energy flux from the brane into the bulk. The simplest solution of this type is the five-dimensional generalization of Vaidya's metric. In this way the brane trajectory, and thus its cosmological evolution, are determined self-consistently via the junction conditions, taking into account possible backreaction effects. The price to be paid for this is the necessity to make the assumption that the trajectories followed by the bulk gravitons are exactly perpendicular to the brane, assumption that is realistic only in the low energy regime.

Although these two approaches are very different, they give estimates that agree within one order of magnitude. Moreover, these estimates are very close to the current observational bound on the number of extra relativistic degrees of freedom. Therefore, it is important to get an accurate determination of the amount of dark radiation expected for this model. Indeed, the improvement of observational constraints on the number of nonstandard relativistic degrees of freedom will allow us to put constraints on the cosmological evolution of the Randall-Sundrum model (possibly excluding a long era of nonstandard $H^2 \propto \rho^2$ cosmology) and on the parameter space of the model. A detailed analysis can also allow us to quantify the effects of the bounces of gravitons on the brane and to estimate to which extent backreaction effects can be relevant. The present analysis, finally, can be generalized to the case of models with extra matter in the bulk [9,10], eventually leading to stronger constraints.

Our plan is the following. In the next section, we discuss the problem from an effective four-dimensional point of view. In Sec. III, we rederive the equations governing the cosmology of the brane and the trajectory of the brane in the bulk. Section IV is devoted to the emission of bulk gravitons by brane particles. In Sec. V, we recall our model [8] based on the Vaidya metric. In Sec. VI, we discuss the propagation of the gravitons in the bulk. In Sec. VII, we present and discuss our numerical computations. We conclude in the final section.

II. EFFECTIVE APPROACH

As a starting point, we will discuss the cosmology of the brane and the influence of the bulk gravitons from a four-dimensional effective description. In order to do so, we will follow the approach of [11].

We start with the 5-dimensional Einstein equations

$$R_{AB}-\frac{1}{2}g_{AB}R+\Lambda_5 g_{AB}=\kappa^2[\mathcal{T}_{AB}+S_{AB}\delta(y)], \tag{4}$$

where the matter component, on the right hand side, consists of a bulk energy-momentum tensor $\mathcal{T}_{AB}$ and distributional brane energy-momentum tensor. The brane is located at $y=0$, y corresponding to the proper coordinate normal to the brane. We assume that the extra dimension has a Z_2 orbifold symmetry and that $y=0$ is a fixed point under this symmetry.

In the cosmological extension of the Randall-Sundrum model [4], S_{AB} is the sum of a brane tension σ, defined by

$$\kappa^2\,\sigma=\sqrt{-6\Lambda}=6\mu, \tag{5}$$

and of the contribution of ordinary matter fields confined to the brane τ_{AB}, i.e.

$$S_{AB}=-\,\sigma\,h_{AB}+\tau_{AB}, \tag{6}$$

where h_{AB} is the metric induced on the brane.

From the above five-dimensional Einstein's equation, using the Z_2 symmetry, one can derive [11] effective four-dimensional equations that read

$${}^{(4)}G_{\mu\nu}=\kappa_4^2(\tau_{\mu\nu}+\tau_{\mu\nu}^{(\pi)}+\tau_{\mu\nu}^{(W)}+\tau_{\mu\nu}^{(B)}), \tag{7}$$

with $\kappa_4^2=\kappa^2\mu$ and

$$\kappa_4^2\tau_{\mu\nu}^{(\pi)}=-\frac{\kappa^2}{24}[6\,\tau_{\mu\alpha}\tau_\nu^\alpha-2\,\tau\tau_{\mu\nu}-h_{\mu\nu}(3\,\tau_{\alpha\beta}\tau^{\alpha\beta}-\tau^2)],$$

$$\kappa_4^2\tau_{\mu\nu}^{(W)}=-{}^{(5)}C_{ABCD}n^An^Bh_\mu^Bh_\nu^D,$$

$$\kappa_4^2\tau_{\mu\nu}^{(B)}=\frac{2\kappa^2}{3}\left[\mathcal{T}_{AB}h^A{}_\mu h^B{}_\nu+h_{\mu\nu}\left(\mathcal{T}_{AB}n^An^B-\frac{1}{4}\mathcal{T}^A{}_A\right)\right], \tag{8}$$

where n^A is the unit vector pointing outward and normal to the brane. In addition to the usual four-dimensional matter energy-momentum tensor $\tau_{\mu\nu}$, three new terms appear on the right hand side of the effective Einstein equations: the first one is quadratic in $\tau_{\mu\nu}$; the second one is the projection on the brane of the 5-dimensional Weyl tensor ${}^{(5)}C_{ABCD}$; and the last one is the projected effect of the bulk energy-momentum tensor.

Since we are interested here in homogeneous brane cosmology, all the effective energy-momenta defined above have necessarily a perfect fluid structure, i.e.

$$\tau_{\mu\nu}^{(i)}=(\rho^{(i)}+p^{(i)})u_\mu u_\nu+p^{(i)}h_{\mu\nu}, \tag{9}$$

where u^μ is the timelike unit vector associated with comoving observers on the brane.

It is then not difficult to show that Eq. (7) gives a Friedmann equation on the brane of the form

$$H^2=\frac{\kappa_4^2}{3}\left[\left(1+\frac{\rho}{2\,\sigma}\right)\rho+\rho^{(W)}+\rho^{(B)}\right], \tag{10}$$

where $H=\dot{a}/a$ is the Hubble parameter on the brane (a is the scale factor on the brane and the overdot stands for a derivative with respect to the cosmic proper time t). We have used the relation $\rho^{(\pi)}=\rho^2/2\sigma$.

Let us now discuss the various (non) conservation laws satisfied by the different energy density components defined above. We follow here the recent analysis of [10], where the bulk energy-momentum tensor was associated to a five-dimensional scalar field. One can first show that the usual conservation equation for cosmological matter is modified into

$$\dot{\rho}+3H(\rho+p)=2\mathcal{T}_{RS}n^R u^S. \tag{11}$$

The right-hand side (evaluated at the brane position) represents the energy flux *from* the bulk *into* the brane. When the brane *loses* energy, as will be the case here via emission of gravitons, the right-hand side is *negative*. The factor 2 is a consequence of the Z_2 symmetry.

Moreover, the 4-dimensional Bianchi identities imply that the total energy density defined by $\rho^{(\mathrm{tot})}=\rho+\rho^{(\pi)}+\rho^{(B)}+\rho^{(E)}$ satisfies

$$\dot{\rho}^{(\mathrm{tot})}+4H\rho^{(\mathrm{tot})}+H\tau^{(\mathrm{tot})\mu}{}_{\mu}=0. \tag{12}$$

Using $\tau^{(\pi)\mu}{}_{\mu}=(\rho/\sigma)(\tau^{\mu}_{\mu}+2\rho)$, $\tau^{(B)\mu}_{\mu}=2(\kappa^2/\kappa_4^2)\mathcal{T}_{AB}n^A n^B$, $\tau^{(W)\mu}{}_{\mu}=0$ and the (non) conservation equation (11), it follows that the energy density for the *dark component*, by which we mean the sum of the components depending explicitly on the bulk, i.e. $\rho_{\mathrm{D}}=\rho^{(B)}+\rho^{(W)}$, satisfies

$$\dot{\rho}_{\mathrm{D}}+4H\rho_{\mathrm{D}}=-2\left(1+\frac{\rho}{\sigma}\right)\mathcal{T}_{AB}u^A n^B-2\frac{H}{\mu}\mathcal{T}_{AB}n^A n^B, \tag{13}$$

where the right hand side is evaluated at the brane position. On the right-hand side of the above equation, one recognizes in the first term the energy flux *from* the brane *into* the bulk, $-2\mathcal{T}_{AB}u^A n^B$. This means, not surprisingly, that the loss of energy inside the brane, will contribute to an *increase* of the amount of dark radiation. In the second term, the quantity $\mathcal{T}_{AB}n^A n^B$ can be interpreted as the *pressure* transverse to the brane, due to the bulk component. In the case of a gas of gravitons, which we consider here, this pressure is positive and therefore this term tends to *decrease* the amount of dark radiation. The two terms on the right hand side have thus opposite effects.

III. COSMOLOGY OF A BRANE IN ADS

We now consider explicitly the bulk and study the trajectory of a brane with relativistic matter in a strictly AdS spacetime, with the metric

$$ds^2=-f(r)dT^2+\frac{dr^2}{f(r)}+r^2 d\mathbf{x}^2,\quad f(r)=\mu^2 r^2. \tag{14}$$

The trajectory of the brane can be represented in terms of its coordinates $T(t)$ and $r(t)$ as functions of the proper time t, which is also the cosmic time in the brane. The normalization of the velocity vector $u^A=(\dot{T},\dot{r},\mathbf{0})$ then implies

$$u^A=\left(\frac{\sqrt{f+\dot{r}^2}}{f},\dot{r},\mathbf{0}\right). \tag{15}$$

One then needs the junction conditions for the brane,

$$[h_A^C\nabla_C n_B]=\kappa^2\left(\tau_{AB}-\frac{1}{3}\tau h_{AB}\right), \tag{16}$$

where n^A is the unit vector normal to the brane (pointing outwards) and is given by

$$n^A=-\left(\frac{\dot{r}}{f},\sqrt{f+\dot{r}^2},\mathbf{0}\right). \tag{17}$$

The spatial components of the junction equations yield the brane Friedmann equation, which can be expressed as

$$\frac{H^2}{\mu^2}=2\frac{\rho}{\sigma}+\frac{\rho^2}{\sigma^2}. \tag{18}$$

It is convenient to introduce the dimensionless quantities

$$\tilde{H}=\frac{H}{\mu},\quad \tilde{\rho}=\frac{\rho}{\sigma}. \tag{19}$$

Note, using Eq. (5) and $\kappa_4^2=\kappa^2\mu$ that the brane tension can be expressed as

$$\sigma=6\mu^2 m_P^2, \tag{20}$$

where $m_P=1/\sqrt{\kappa_4}$ is the reduced Planck mass. We will drop the tildes from now on.

In terms of H, the components of the brane velocity and of the normal vector read respectively

$$u^A=\left(\frac{1}{r}\sqrt{1+H^2},rH,\mathbf{0}\right),\quad n^A=-\left(\frac{H}{r},r\sqrt{1+H^2}\right). \tag{21}$$

The trajectory of the brane is obtained by solving

$$\frac{dr}{dT}=\frac{\dot{r}}{\dot{T}}=r^2\frac{H}{\sqrt{1+H^2}}=r^2\frac{\sqrt{2\rho+\rho^2}}{1+\rho}, \tag{22}$$

where the second equality follows from Eq. (21) and the third from Eq. (18). In the case of radiation domination, $\rho=\rho_i r^{-4}$, where we have defined the radial coordinate r such that $r=1$ for $\rho=\rho_i$, where ρ_i is the energy density at some fiducial initial time t_i. To get the brane trajectory, one must integrate

$$\frac{dr}{dT}=r^2\frac{\sqrt{2\rho_i r^4+\rho_i^2}}{r^4+\rho_i}. \tag{23}$$

Explicit integration is possible in the high energy regime, $\rho_i/r^4 \gg 1$, where one finds

$$\frac{1}{r} \simeq -T + \text{const} \tag{24}$$

and in the low energy regime, where

$$r \simeq \sqrt{2\rho_i}\; T + \text{const.} \tag{25}$$

In the regime of transition between the high energy phase and the low energy phase, i.e. for $r \sim \rho_i^{-1/4}$, one must resort to numerical integration.

IV. EMISSION OF BULK GRAVITONS

We now consider the production of bulk gravitons by the cosmologically evolving brane. In particular, we wish to compute explicitly the components of the bulk energy-momentum tensor due to the gas of gravitons emitted by the brane.

Let us first recall that the energy-momentum tensor due to a gas of massless particles is given by

$$\mathcal{T}_{AB} = \int d^5p \;\; \delta(p_M p^M) \sqrt{-g} f p_A p_B , \tag{26}$$

where $f = f(x^A, p_A)$ is the distribution function of the gravitons (which is a scalar that depends on the spacetime position and on the momentum).

Since the gravitons living in the bulk will be assumed to have been produced only by emission from the brane, it will be convenient to start the computation of the energy-momentum tensor components on the trajectory of the brane. At the location of the brane, we can see the bulk gravitons from two perspectives.

First, from the brane point of view, the bulk gravitons are seen as massive four-dimensional particles with mass m, three-momentum $\mathbf{p}$ and energy E, satisfying $E = \sqrt{\mathbf{p}^2 + m^2}$. They are created by the scattering of two ordinary particles confined on the brane. The leading contribution to this process is given by the scattering $\psi\,\bar{\psi} \rightarrow$ graviton, where ψ is a standard model particle. At the cosmological level, the production of gravitons results into an energy loss for ordinary matter, which can be expressed as

$$\frac{d\rho}{dt} + 3H(\rho + P) = -\int \frac{d^3p}{(2\pi)^3} \mathbf{C}[f], \tag{27}$$

with

$$\mathbf{C}[f] = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \sum |\mathcal{M}|^2 f_1 f_2 \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p), \tag{28}$$

where $\mathcal{M}$ is the scattering amplitude for the process in consideration, and the indices 1 and 2 correspond to the scattering particles (ψ and $\bar{\psi}$).

Second, from the bulk point of view, the gravitons are massless particles, each graviton being characterized by a five-dimensional momentum, which can be decomposed into a spatial four-momentum and an energy, defined with respect to a reference frame.

To make the connection between these two points of view, it is convenient to choose a reference frame associated to a (comoving) brane observer. Introducing an orthonormal frame defined by u^A, n^A and e_i^A, the five-dimensional momentum of any graviton can be decomposed into

$$p^A = E u^A + m n^A + \sum_i \tilde{p}_i e_i^A . \tag{29}$$

The components along the vectors e_i^A are denoted with a tilde to distinguish them from the components p^i defined in the decomposition along the coordinate vectors $(\partial/\partial x^i)^A$. If one substitutes this decomposition into Eq. (26), one gets for the mixed component

$$\mathcal{T}_{nu} \equiv u^A n^B \mathcal{T}_{AB} = \int d^5p\, \delta(p_M p^M) \sqrt{-g}\, f p_A p_B n^A u^B = -\int dm d^3\mathbf{p}\, \frac{m}{2} f. \tag{30}$$

By identifying the right hand sides of Eqs. (11) and (27), one immediately finds, upon comparing Eq. (28) with the above expression, that the distribution function at the brane location, for gravitons which are being emitted, is given by

$$f_{(em)}(m, \mathbf{p}) = \frac{1}{2m} \frac{1}{(2\pi)^5} \int \frac{d^3\mathbf{p}_1}{2p_1} \frac{d^3\mathbf{p}_2}{2p_2} \sum |\mathcal{M}|^2 \times f_1 f_2 \delta^{(4)}(p_1 + p_2 - p). \tag{31}$$

The summed squared amplitude is given by the expression

$$\sum |\mathcal{M}|^2 = A \frac{\kappa^2}{8\pi} s^2 \tag{32}$$

where s is the Mandelstam invariant and

$$A = \frac{2}{3} g_s + g_f + 4 g_v , \tag{33}$$

where g_s, g_f and g_v are respectively the scalar, fermion and vector relativistic degrees of freedom in thermal equilibrium on the brane. For the standard model, if all degrees of freedom are relativistic, one has $g_s = 4$, $g_f = 90$ and $g_v = 24$. For simplicity, we will ignore the Bose or Fermi corrections in the distribution functions and assume that the particles on the brane are characterized by a Boltzmann distribution of temperature T. The distribution function at emission, $f_{(em)}$, can then be computed explicitly and one finds (neglecting the masses of the scattering particles)

$$f_{(em)}(m, \mathbf{p}) = \frac{A}{2^{10}\pi^5} \kappa^2 m^3 e^{-\sqrt{\mathbf{p}^2 + m^2}/T}. \tag{34}$$

We now have all the elements to compute all the components of the bulk energy-momentum tensor due to the emitted gravitons at the brane location. They are given by

$$\mathcal{T}_{uu}^{(em)}=\int dmd^3\mathbf{p}\frac{E}{2}f_{(em)}=\frac{21}{16\pi^4}A\kappa^2T^8=\frac{4725}{4\pi^8}\frac{A}{g_*^2}\kappa^2\rho^2, \tag{35}$$

$$\mathcal{T}_{un}^{(em)}=-\int dmd^3\mathbf{p}\frac{m}{2}f_{(em)}=-\frac{315A}{2^{10}\pi^3}\kappa^2T^8 = -\frac{70875}{2^8\pi^7}\frac{A}{g_*^2}\kappa^2\rho^2, \tag{36}$$

$$\mathcal{T}_{nn}^{(em)}=\int dmd^3\mathbf{p}\frac{m^2}{2E}f_{(em)}=\frac{3A}{4\pi^4}\kappa^2T^8=\frac{675}{\pi^8}\frac{A}{g_*^2}\kappa^2\rho^2, \tag{37}$$

with $E\equiv\sqrt{\mathbf{p}^2+m^2}$ as stated before. In the second equalities, we have replaced the temperature by the energy density, using $\rho=(\pi^2/30)g_*T^4$, where g_* is the effective number of relativistic degrees of freedom. The above components can be interpreted physically respectively as the energy density, the energy flux and the lateral pressure of the emitted bulk gravitons as measured by an observer at rest with respect to the brane.

V. COSMOLOGICAL EVOLUTION IN THE VAIDYA MODEL

In this section, we deviate from the preceding analysis and consider the evolution of the dark radiation in the context of the Vaidya model introduced in [8] (see also [12]). The interest of the Vaidya model is to work with an explicit five-dimensional realization of the effective equations introduced in Sec. II, where there is a non zero energy transfer between the brane and the bulk. This however imposes that all the gravitons have to be assumed to be emitted *radially* in the five-dimensional bulk so that the bulk energy-momentum tensor is of the form

$$\mathcal{T}_{AB}=\sigma_B k_A k_B, \tag{38}$$

where k^A is a null vector. Assuming spherical symmetry, Einstein's equations (4) with such an energy-momentum can be solved analytically: this is the Vaidya solution [13], ordinarily used to describe a radiating relativistic star. Its metric is given by

$$ds^2=-f(r,v)dv^2+2dr\,dv+r^2d\vec{x}^2, \tag{39}$$

with

$$f(r,v)=\mu^2r^2-\frac{\mathcal{C}(v)}{r^2}. \tag{40}$$

For a constant $\mathcal{C}$, one recovers, after changing the light-like coordinate v into the static time coordinate t, the familiar five-dimensional AdS-Sch metric.

It is instructive to apply the effective equations derived earlier in this particular context. First, one can normalize the null vector k^A such that $k_Au^A=1$, in which case the projections of the bulk energy-momentum tensor (38) are given by $\mathcal{T}_{nu}=-\sigma_B$ and $\mathcal{T}_{nn}=\sigma_B$. As a consequence the nonconservation equation (11) for the energy density on the brane reads

$$\dot{\rho}+3H(\rho+p)=-2\sigma_B, \tag{41}$$

and σ_B, up to the factor of 2 due to the Z_2 symmetry, directly represents the energy loss in the brane due to the production of bulk gravitons. The dependence of σ_B is given explicitly in Eq. (36), and in terms of the brane energy density,

$$\sigma_B\propto\rho^2. \tag{42}$$

The Weyl parameter $\mathcal{C}$ can be related to the "dark" component energy density defined earlier via

$$\rho_D=\frac{\mathcal{C}}{a^4}. \tag{43}$$

Substituting in the evolution equation (13), one gets [still using the implicit tilded quantities defined in Eq. (19)]

$$\frac{\dot{\mathcal{C}}}{a^4}=-2(1+\rho)\mathcal{T}_{AB}u^An^B-2H\mathcal{T}_{AB}n^An^B. \tag{44}$$

With the explicit bulk energy-momentum tensor (38), $T_{nu}=-\sigma_B$ and $T_{nn}=\sigma_B$, Eq. (44) reduces to

$$\frac{\dot{\mathcal{C}}}{a^4}=2\sigma_B(1+\rho-H), \tag{45}$$

where one recognizes the result already derived in [8], but there from Einstein's equations rather than from the effective equations like here. The Friedmann equation, when one neglects the dark radiation contribution, gives during the high energy regime

$$H\simeq\rho+1-\frac{1}{4\rho}. \tag{46}$$

This shows that there is a remarkably precise compensation, *at leading order and next to leading order*, between the energy flux and the transverse pressure in the dark radiation equation. And the production of $\mathcal{C}$ is governed by

$$\frac{\dot{\mathcal{C}}}{a^4}\simeq\frac{\sigma_B}{2\rho}\propto\rho. \tag{47}$$

As mentioned above, the drawback of the Vaidya description is the assumption that all gravitons are radial. As seen in the previous section, the distribution of emitted gravitons is not radial and $\mathcal{T}_{nn}+\mathcal{T}_{un}$ is not zero. Therefore, if one substitutes the explicit values of $\mathcal{T}_{nn}$ and $\mathcal{T}_{un}$ in Eq. (44), we no longer get the very precise compensation observed in the

Vaidya case and the global sign is positive [since $315/(2^{10}\pi^3) > 3/(4\pi^4)$], so that the production of dark radiation seems to be driven in the high energy regime by a term on the right hand side proportional to ρ^3, rather than proportional to ρ as in the Vaidya description. Obviously, this would result in an enormous amount of dark radiation, far above the estimate of [8], and this would ruin the simplest brane cosmology scenario, since the estimate of [8] was barely within the nucleosynthesis bounds.

The above analysis is however incomplete. In the Vaidya description, the gravitons are emitted radially inwards and are therefore lost for the brane once there are emitted. Some of the non radial gravitons, however, can come back onto the brane after their emission, as it will be shown in the next section, and thus influence once more the evolution of the Weyl parameter. Because of the Z_2 symmetry, these gravitons will be reflected by the brane (we ignore here the decay of a bulk graviton into brane particles) and will contribute only to the transverse pressure term. The effect of these old gravitons will thus be to reduce the amount of dark radiation that would be computed naively by considering only gravitons being emitted.

VI. GRAVITON TRAJECTORIES IN THE BULK

After the gravitons are emitted, they move freely in the bulk, each individual graviton following a null geodesic. The null geodesics in five-dimensional AdS were studied in [14]. We summarize here the main results. Using the symmetries of the metric (14), one can identify the first integrals for the geodesic motion

$$f(r)\frac{dT}{d\lambda} = \mathcal{E}, \quad r^2\frac{dx^i}{d\lambda} = \mathcal{P}^i, \tag{48}$$

where λ is any affine parameter. For any graviton one can choose the affine parameter so that the tangent vector of the null trajectory is identified with the physical momentum p^A. Introducing the notation $\tilde{p}^T = r p^T$ and $\tilde{p}^r = p^r/r$, the above first integrals become

$$\mathcal{E} = r\tilde{p}^T \tag{49}$$

and

$$\boldsymbol{\mathcal{P}} = r\mathbf{p}. \tag{50}$$

Using these conservation laws, it is easy to determine the trajectory of the gravitons in the bulk spacetime. In order to do so, one can compute

$$\frac{dr}{dT} = \left(\frac{dr}{d\lambda}\right) \Big/ \left(\frac{dT}{d\lambda}\right). \tag{51}$$

Since $p^A p_A = 0$, one gets

$$(p^r)^2 = \mathcal{E}^2 - \mathcal{P}^2, \tag{52}$$

which means that p^r is also a constant of motion. It is then useful to introduce the parameter

$$\mathcal{V} = \frac{p^r}{\mathcal{E}}, \tag{53}$$

which will be constant along the null geodesic trajectory, and in terms of which the trajectory in the extra dimension is given by

$$\frac{dr}{dT} = \mathcal{V} r^2. \tag{54}$$

Integration gives the graviton trajectory

$$T - T_* = -\frac{1}{\mathcal{V}}\left(\frac{1}{r} - \frac{1}{r_*}\right). \tag{55}$$

It is also useful to relate the bulk-based description used above with the brane-based approach introduced earlier. Using the decomposition (29) with the velocity and normal vectors given in Eq. (21), one finds

$$\tilde{p}^T = \sqrt{1+H^2}E - Hm, \quad \tilde{p}^r = HE - \sqrt{1+H^2}m. \tag{56}$$

Inverting this system, to get

$$m = H\tilde{p}^T - \sqrt{1+H^2}\,\tilde{p}^r, \quad E = \sqrt{1+H^2}\tilde{p}^T - H\tilde{p}^r. \tag{57}$$

For some values of the parameters, the trajectory of the graviton, leaving the brane at the emission point, can cross again the brane trajectory. It will then be reflected by the brane, as a consequence of the Z_2 symmetry.

It is not difficult to compute the new five-dimensional momentum of the graviton after the reflection on the brane, by considering the reflection from the brane point of view. The momentum parallel to the brane $\mathbf{p}$ is conserved, as well as the energy E. Only the transverse momentum is affected and simply changes its sign. The momentum along the spatial direction orthogonal to the brane is embodied by the mass m. We will adopt the convention that m is positive when the momentum is outwards, with respect to the brane, and negative otherwise. The reflection of the graviton by the brane is thus governed by the simple laws,

$$E \to E, \quad \mathbf{p} \to \mathbf{p}, \quad m \to -m. \tag{58}$$

In the bulk-based point of view, this translates into

$$\tilde{p}^T_{out} = (1+2H^2)\tilde{p}^T_{in} - 2H\sqrt{1+H^2}\,\tilde{p}^r_{in}, \tag{59}$$

$$\tilde{p}^r_{out} = 2H\sqrt{1+H^2}\tilde{p}^T_{in} - (1+2H^2)\tilde{p}^r_{in}. \tag{60}$$

We can use the previous results to infer the evolution of the graviton distribution function throughout the bulk. In principle, it is governed by the Liouville equation, which, in general relativity, reads

$$p^A\frac{\partial f}{\partial x^A} + \Gamma^C_{AB}p^A p^B\frac{\partial f}{\partial p^C} = 0. \tag{61}$$

However, this equation is in fact no more than the implementation at the level of the distribution function of the fact that each individual particle follows a geodesic. To solve explicitly the Liouville equation it is therefore simpler to use directly the solutions for the geodesic trajectories.

This enables us to write the emission distribution function, at the brane location, in terms of the bulk-based momenta, by simply substituting the above expressions in Eq. (34). Moreover, using the constants of motion along the graviton geodesics established above, one can deduce the expression for the distribution function off the brane as well. One gets

$$f_{em}^{bulk}(T,r,\tilde{p}^T,\tilde{p}^r,\mathbf{p})=f_{(em)}\left(T_{em},a_{em},\frac{r}{a_{em}}\tilde{p}^T,\frac{r}{a_{em}}\tilde{p}^r,\frac{r}{a_{em}}\mathbf{p}\right), \tag{62}$$

where the time and brane scale factor at emission depend on t, r, $\tilde{p}^T$ and $\tilde{p}^r$ and are obtained by tracing the graviton geodesic back in time until it crosses the brane trajectory. The rescaling of the momenta on the right hand side are just a consequence of the conservation laws (49) and (50) (the same rescaling for $\tilde{p}^r$ follows from the normalization of the five-dimensional momentum). By construction, the above distribution function must be a solution of the Liouville equation (61) in the bulk, as one can explicitly check.

The above expression describes gravitons which have been emitted by the brane and have not returned back onto the brane. But, as already mentioned, the situation is in fact more complicated because some of the gravitons that have been emitted by the brane can come back onto the brane and be reflected back into the bulk with a different momentum. Therefore, at each point along the brane trajectory, one must add to the gravitons that are being emitted all the older gravitons that are being reflected by the brane at the same instant.

Actually, the *majority* of gravitons emitted in the high energy regime will be reflected at least once by the brane. In the period in which $H^2\propto\rho^2$ (i.e. $H\gg1$) the brane is indeed moving relativistically with respect to the frame defined by the metric (14). The gravitons that are not emitted exactly orthogonal to the brane will see their radial momentum boosted, and will thus move in this frame in the same direction as the brane. This can be seen by inspection of Eq. (56): for $H\gg1$ and E not too close to m, p^r is positive. As H decreases, the brane will eventually slow down, and the gravitons will bounce off it.

Let us consider a fiducial time, characterized by t_0, in the history of the brane. In order to compute the increment of the Weyl parameter, via Eq. (44), one must add to the newborn gravitons, which are just being emitted, the old gravitons that are being reflected by the brane. For these old gravitons, not surprisingly, the net contribution to the energy flux T_{nu} vanishes, since they are just reflected, neither absorbed nor created, whereas the incoming and outgoing contributions to the pressure are equal in magnitude and opposite in sign. For these gravitons, therefore, we need to compute only the component

$$T_{nn}^{(in)}=\int dmd^3\mathbf{p}\frac{m^2}{2E}f_{(in)}. \tag{63}$$

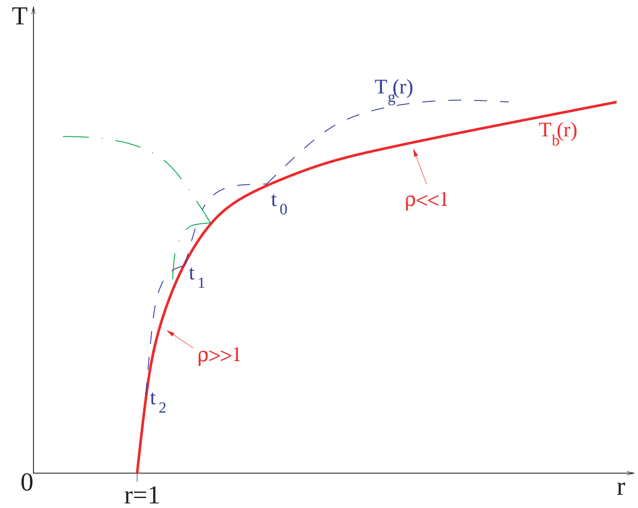

FIG. 1. Trajectory of the brane and typical trajectories of gravitons in the metric (14). The brane trajectory is the solid thick line, the steepest part on the left corresponding to the high energy regime $\rho\gg1$. The dashed line describes a graviton produced at t_2, bouncing off the brane at t_1 and again at t_0. The dashed-dotted line corresponds to a second graviton, that, after being reflected once by the brane, falls into the bulk.

Let us now write explicitly the contribution to f_{in} from gravitons which make at the time t_0 their first bounce since their emission. It will be convenient to introduce the following parametrization

$$H=\sinh\beta, \tag{64}$$

for the Hubble rate, and for each graviton, the parameter x such that

$$E=p\cosh x,\quad m=p\sinh x, \tag{65}$$

where $p\equiv|\mathbf{p}|$ is the norm of the three-momentum parallel to the brane. Using the results obtained previously, one finds that the energy E_1 and mass m_1 measured in the brane frame at the time of emission t_1 are given by the simple expressions

$$E_1=\frac{a_0}{a_1}p\cosh(\beta_1-\beta_0+x),$$

$$m_1=\frac{a_0}{a_1}p\sinh(\beta_1-\beta_0+x), \tag{66}$$

where

$$\beta_1=\beta_1(\beta_0,x) \tag{67}$$

is determined by tracing backwards the null geodesic (see Fig. 1) that intersects the brane trajectory at t_0 until the previous intersection. More explicitly, β_1 is obtained by solving the equation $T_b(r)=T_g(r)$, where $T_b(r)$ gives the time T as a function of the position r of the brane through Eq. (23), whereas $T_g(r)$ is given by the graviton geodesic equation (55).

Substituting

$$^{(1)}f_{(in)}=\frac{A}{2^{10}\ \pi^5}\kappa^2 m_1^3 e^{-E_1/T_1}, \tag{68}$$

in the integral (63), one finds, after integration over p,

$$^{(1)}\mathcal{T}_{nn}^{(in)}(\beta_0)=\frac{7!A}{2^9\pi^4}\kappa^2\int dx\left(\frac{a_0}{a_1}\right)^{-5}T_1^8 \times\frac{\sinh^2 x\sinh^3(\beta_1(x)-\beta_0+x)}{\cosh^8(\beta_1(x)-\beta_0+x)}. \tag{69}$$

In a similar way, one can compute the fraction $^{(2)}f_{in}$ due to the gravitons for which this is the second bounce on the brane since their emission. Consider such a graviton, which was emitted at time t_2, was reflected by the brane once at time t_1 and comes back again onto the brane at time t_0 ($t_2<t_1<t_0$). Just before the first bounce, the extra-dimensional momentum is given by

$$m_1^{(in)}\equiv p_1\sinh x_1=-m_1^{(out)}=-\frac{a_0}{a_1}p\sinh(\beta_1-\beta_0+x). \tag{70}$$

Hence, at emission, we had

$$m_2^{(out)}=\frac{a_0}{a_2}p\sinh(\beta_2-2\beta_1+\beta_0-x) \tag{71}$$

and thus

$$E_2^{(out)}=\frac{a_0}{a_2}p\cosh(\beta_2-2\beta_1+\beta_0-x). \tag{72}$$

Consequently, the contribution to the transverse pressure is of the form

$$^{(2)}\mathcal{T}_{nn}^{(in)}(\beta_0)=\frac{7!A}{2^9\pi^4}\kappa^2\int dx\left(\frac{a_0}{a_2(x)}\right)^{-5}T_2^8(x) \times\frac{\sinh^2 x\sinh^3(\beta_2(x)-2\beta_1(x)+\beta_0-x)}{\cosh^8(\beta_2(x)-2\beta_1(x)+\beta_0-x)} \tag{73}$$

where we have stressed the dependence on x in the integrand (there is also a dependence on β_0). It is then straightforward to generalize to the contribution of gravitons with any number of bounces.

VII. NUMERICAL RESULTS

A. Source terms

Whereas the contributions from the newborn gravitons to the energy flux and pressure can be computed analytically, one must resort to numerical integration to compute the contribution from old gravitons. For the latter case, we need only the contribution to the pressure, since the contribution to the energy flux vanishes as explained earlier. Therefore, one can write the evolution equation for the dark component as

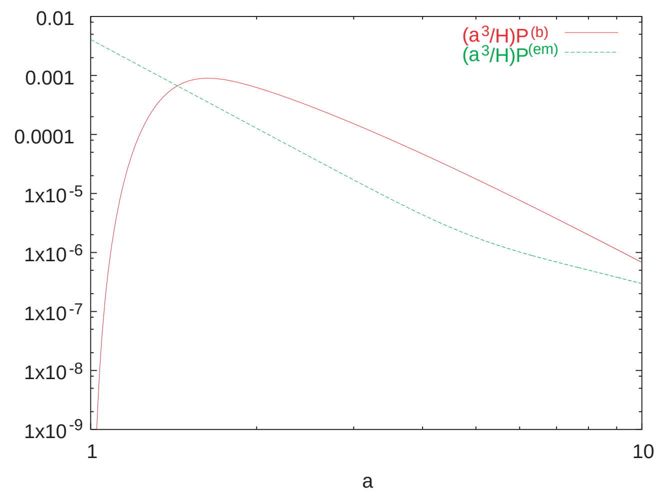

FIG. 2. Contributions to the emission of dark radiation (for $\rho_i=1000$) from graviton emission and from the pressure of bulk gravitons. $\mathcal{P}^{(b)}$ and $\mathcal{P}^{(em)}$ are defined in Eq. (75).

$$\dot{\rho}_D+4H\rho_D=\mathcal{P}^{(em)}-\mathcal{P}^{(b)}, \tag{74}$$

with

$$\mathcal{P}^{(em)}=-2(1+\rho)\mathcal{T}_{un}^{(em)}-2H\mathcal{T}_{nn}^{(em)},\quad \mathcal{P}^{(b)}=4H\mathcal{T}_{nn}^{(in)}. \tag{75}$$

The above equation can be rewritten as

$$\frac{d}{da}(a^4\rho_D)=\frac{a^3}{H}\mathcal{P}^{(em)}-\frac{a^3}{H}\mathcal{P}^{(b)}, \tag{76}$$

where $\mathcal{P}^{(em)}$ can be computed analytically by means of Eqs. (36), (37).

We have computed numerically the function $\mathcal{P}^{(b)}(a)$ using the generalization of Eq. (73), taking into account gravitons that have made up to 10^3 bounces before hitting the brane when its scale factor was a. We have checked that neglecting the effect of gravitons that have made more than 10^3 bounces does not change appreciably our results. We have also assumed that the bulk contains no gravitons at the initial time t_i.

Numerically reliable results could be obtained for values up to $\rho_i\simeq 10^3$. Taking the lowest value of μ compatible with the data (from small-scale gravity experiments), $\mu^{-1}\sim 0.1$ mm, one gets that the brane tension has to be at least of the order of the TeV, which corresponds to a fundamental Planck scale $M_5\equiv\kappa^{-2/3}\simeq 10^8$ GeV. As a consequence, the highest initial energy density which makes sense is $\rho_i\sim M_5^4/\sigma\sim 10^{20}$.

In order to compare, at each instant in the history of the brane, the contribution directly due to the emission of gravitons with the one due to the reflection of older gravitons, we have plotted in Fig. 2 the two terms on the right hand side of Eq. (76). One observes that at early times the dominant con-

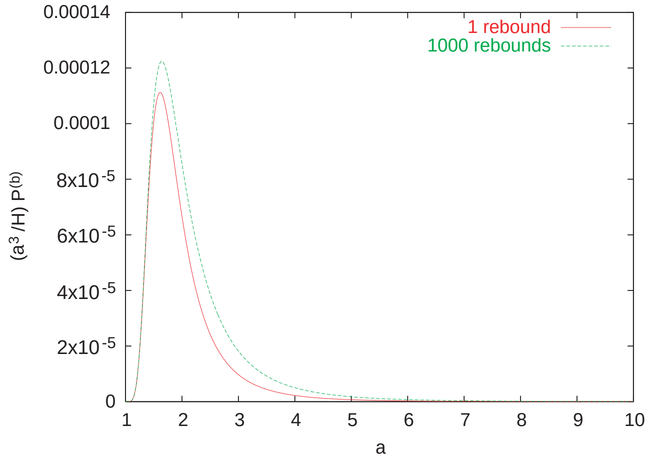

FIG. 3. Contribution to the source term $(a^3/H)\ \mathcal{P}^{(b)}$ from gravitons that have been reflected only once by the brane, with respect to the total term (numerically, up to 10^3 bounces).

tribution is that due to the emission of gravitons, which is not surprising since the bulk is assumed to be empty initially. The bulk is then gradually filled with gravitons and some of them can come back onto the brane and be reflected. They contribute to the transverse pressure effect, which can be seen on the plot to build very quickly. In the intermediate phase, the pressure effect dominates the emission effect so that the net source term for the dark radiation is negative. However, the two *integrated* effects are very close in amplitude, which means that the compensation observed in the simple Vaidya model is still working in this case. This is also the origin of the numerical difficulties of the present analysis: the result we are looking for is the small difference of two very large numbers. One can indeed see that both terms on the right hand side of Eq. (76) scale roughly as ρ_i^2, whereas their difference scales approximately as ρ_i. Notice that, for this reason, the quantities on the vertical axes of Fig. 2 and 3 have been rescaled by a factor ρ_i^2.

It is also important to stress the necessity to take into account the multiple reflections of gravitons on the brane to get a correct evaluation of the total effect. To illustrate this point, we have plotted in Fig. 3 the contribution from gravitons for which this is the first bounce in comparison with the cumulative contribution from gravitons that have made up to 10^3 bounces.

B. Dark radiation

Our main goal is to compute the dark radiation globally produced in the process. At very low energy, dark radiation is produced at a negligible rate, so that one can consider that there is an asymptotic constant value for the Weyl parameter $\mathcal{C}$. In the end, this asymptotic value for $\mathcal{C}$ depends only on the initial energy density in the brane and on the number of relativistic degrees of freedom during the high energy phase. It will be convenient to express the final amount of dark radiation as its ratio with respect to standard radiation energy density

$$\epsilon_D \equiv \frac{\rho_D}{\rho_{rad}}. \tag{77}$$

Such quantity is constrained by cosmological observations. The amount of nonstandard radiation in the early universe is usually measured in units of extra neutrino species ΔN_ν. The conversion factor between ΔN_ν and ϵ_D is $\Delta N_\nu = (43/7)(g_*^{\rm nucl}/g_*)^{1/3}\ \epsilon_D$, where $g_*^{\rm nucl} = 10.75$ is the number of relativistic degrees of freedom just before nucleosynthesis, more precisely just before the electron-positon annihilation. The factor $(g_*^{\rm nuc}/g_*)^{1/3}$ accounts for the change in the number of relativistic degrees of freedom since the era in which dark radiation has been produced. Assuming $g_* = 106.75$, this gives $\epsilon_D \simeq 0.35\ \Delta N_\nu$. Constraints on ΔN_ν depend upon the kind of observation one takes into account, but a typical order of magnitude is $\Delta N_\nu < 0.2$ [15,16], which gives an upper limit $\epsilon_D < 0.07$. According to both the analyses [15,16], the favored value of ΔN_ν turns out to be *negative*. Although this possibility is allowed by a negative value of the Weyl parameter $\mathcal{C}$, this would correspond for the metric (1) to a naked singularity in the bulk, a situation that is certainly not appealing from the theoretical point of view.

In the evaluation of the total amount of dark radiation produced, an important quantity is the energy loss due to production of gravitons, which depends on the number of relativistic degrees of freedom confined on the brane. In LSR, the expression for the energy loss is given by

$$\sigma_B \equiv \frac{\alpha}{12}\kappa^2 \rho^2, \tag{78}$$

where

$$\alpha = \frac{212625}{64\pi^7}\zeta(9/2)\zeta(7/2)\frac{\hat{g}}{g_*^2} \tag{79}$$

with

$$\hat{g} = ((2/3)g_s + 4g_v + (1-2^{-7/2})(1-2^{-5/2})g_f),$$
$$g_* = g_s + g_v + (7/8)g_f. \tag{80}$$

This is in exact agreement with the revised version [17] of HM.

In the present work, $\sigma_B = -\mathcal{T}_{un}^{(em)}$ and the expression for $\mathcal{T}_{un}^{(em)}$ is given in Eq. (36) so that α, defined as in Eq. (78), is given by

$$\alpha = \frac{212625}{64\pi^7}\frac{A}{g_*^2}. \tag{81}$$

The difference between the two above values for α comes from the fact that Fermi-Dirac or Bose-Einstein distribution functions were used in HM and LSR, whereas here we have treated all the particles on the same footing and assume simply a Boltzmann distribution. To make a precise comparison between the previous estimates and our numerical results, we will use the analytical estimates of HM and LSR with the

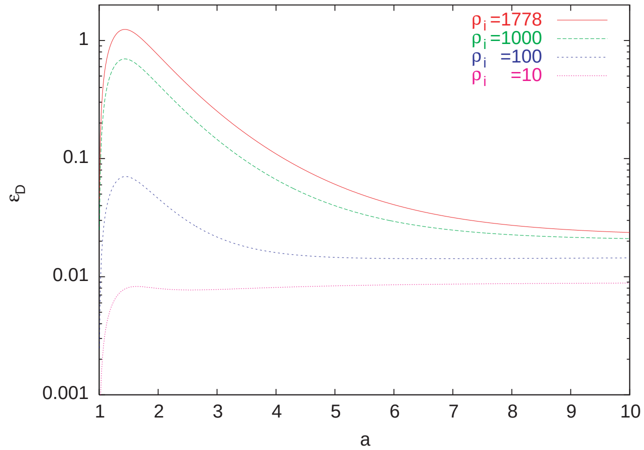

FIG. 4. Evolution of $\epsilon_D = \rho_D/\rho$ for different values of ρ_i.

value for α given in Eq. (81). For the particle content of the standard model, the values of α as given by Eqs. (79) and (81) differ only by about the 5%.

Let us now recall these previous estimates. In HM, the amount of dark radiation produced in the high energy regime can be expressed as

$$\epsilon_D \simeq \mathcal{F}\frac{\alpha}{4}\ln(\rho_i/2), \tag{82}$$

where $\mathcal{F}$ is an efficiency factor (denoted α in HM) with $5\pi/32 < \mathcal{F} < 1$. The amount of dark radiation produced during the whole low energy regime (assumed in HM to start at $\rho = 2$) is $\alpha/2$. Therefore in our comparison we will take for the total fraction of dark radiation as estimated in HM the expression

$$\epsilon_D^{HM} = \frac{\alpha}{4}[2 + \mathcal{F}\ln(\rho_i/2)] \ , \quad \rho_i \gg 2. \tag{83}$$

The value of ϵ_D as given by LSR is obtained by solving a system of differential equations including Eq. (45). In the limit of large ρ_i one gets $\epsilon_D \to \alpha/4$.

Let us now evaluate the amount of dark radiation produced in our numerical approach. Starting from the equation

$$\dot{\rho}_D + 4H\rho_D = \mathcal{P}^{(em)} - \mathcal{P}^{(b)}, \tag{84}$$

where the source term is given explicitly above, one finds

$$\epsilon_D(t_0) = \frac{\rho_D(t_0)}{\rho_{rad}(t_0)} = \frac{1}{\rho_i}\int_{t_i}^{t_0} dt \left(\frac{a(t)}{a_i}\right)^4 (\mathcal{P}^{(em)} - \mathcal{P}^{(b)}). \tag{85}$$

We have integrated numerically this equation and plotted in Fig. 4 the evolution of the ratio $\epsilon_D = \rho_D/\rho$ as a function of the scale factor. Remarkably, for large enough initial values of the energy density on the brane ρ_i, the dark radiation component can dominate the brane matter energy density at early times. This however does not seem to invalidate the implicit assumption that one can neglect the backreaction of bulk gravitons on the bulk geometry. Indeed, the dark energy domination, $\rho_D(a) > \rho(a)$, is effective only in the high energy regime $\rho \gg 1$. And in this regime, the cosmological expansion is dominated by ρ^2, which remains much larger than ρ_D. Nevertheless, it might be interesting to explore the consequences of the fact that in the very early evolution of the Randall–Sundrum universe, most of the energy density of the universe should be in the form of dark radiation (or bulk gravitons) gravitationally bound to the brane.

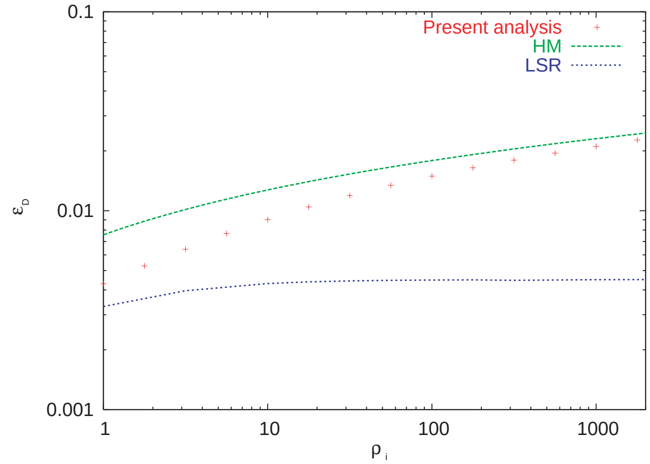

FIG. 5. Comparison of the numerical results of the present work with the estimates of HM (upper curve) and of LSR (lower curve) for the amount of dark radiation $\epsilon_D = \rho_D/\rho$ as a function of the initial energy density on the brane ρ_i.

In Fig. 5, we compare the numerical estimates of the present work with the analytical estimates of HM and LSR. We took for α the expression (81) with all the degrees of freedom of the standard model. The curve corresponding to HM comes from Eq. (83) with the lowest value of $\mathcal{F}$.

Due to the difficulties mentioned above, the range of our numerical analysis is limited to values of ρ_i smaller than about 2×10^3. In this range, we get an accurate description of the behavior of ϵ_D as a function of ρ_i. We see that ϵ_D is a slowly increasing function of ρ_i. At low values of ρ_i, the numerical results are close to the estimate of LSR. Indeed, for $\rho_i \lesssim 1$, the effect of graviton bounces can be neglected, and the Vaidya description analyzed in Sec. V gives a good approximation, whose results agree, in the limit $\rho_i \ll 1$, also with those of HM. Going to higher values of ρ_i, in the regime of validity of our analysis the value of ϵ_D is below the lowest bound estimated by HM, but it gets closer and closer as ρ_i increases and one can expect it to become higher already for ρ_i of the order of few thousands (remember that the maximal value of ρ_i compatible with constraints on the Randall-Sundrum model is $\rho_i \sim 10^{20}$). It is remarkable that already for the (relatively) low value $\rho_i \simeq 1800$, $\epsilon_D > 0.02$, which is not far from the upper bound imposed by CMB and BBN observations [15,16].

VIII. CONCLUSIONS

In the present work, we have computed the amount of dark radiation produced during the cosmological evolution of our brane universe. Previous estimates were based on relatively crude approximations: in one case, the evolution of the brane was separated into a high energy phase and a low energy phase; in the other case, all bulk gravitons were supposed to be radial. However, both estimates were pointing to values very close to the current bound on extra relativistic degrees of freedom, which was motivating a more detailed study of the question.

To go beyond the previous approximations, the present work uses a numerical approach, which enables us to treat smoothly the transition between the high and low energy regimes and to deal with nonradial gravitons. Our initial objective to compute precisely the amount of dark radiation produced is however hampered by a problem of numerical precision, and we have been able to do this computation only for moderate values of the initial brane energy density. The reason for this limitation lies in a remarkable compensation between two opposite effects: the emission of bulk gravitons by the brane, which contributes *positively* to the dark radiation, and the pressure of old gravitons bouncing off the brane, which contributes *negatively* to the dark radiation. Numerically, we have been obliged to compute these two effects separately, and the net effect, in which we are interested, comes from the difference of two quasi-equal huge numbers, which is difficult to control numerically.

With the present analysis, we could estimate the evolution of the dark radiation component as a function of time, finding that in the very early stages of a radiation dominated brane universe, energy as dark radiation could easily exceed the amount of energy in brane matter.

In the range of initial densities where we can rely on our numerical computation, we have been able to compare our result with the analytical estimates obtained previously and our results agree with their order of magnitude.

The increase of the amount of dark radiation produced with the initial energy density on the brane seems to indicate that a significant amount of dark radiation will be produced for an extended high energy (nonstandard) period in the early universe. However, we cannot safely extrapolate the present analysis to higher values of the initial energy density on the brane and we believe that an extension of our numerical computation further into the very high energy regime will tell us whether a long period of nonstandard cosmology is going to be soon ruled out by observations.

ACKNOWLEDGMENTS

We wish to thank T. Tanaka for very stimulating discussions at an early stage of the present work. The work of L.S. is supported by the European Community's Human Potential Program under contract HPRN-CT-2000-00152 Supersymmetry and the Early Universe.

[1] P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
[2] C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999).
[3] P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 2000 (285).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[5] D. Langlois and M. Rodriguez-Martinez, Phys. Rev. D **64**, 123507 (2001).
[6] C. Charmousis, Class. Quantum Grav. **19**, 83 (2002).
[7] A. Hebecker and J. March-Russell, Nucl. Phys. **B608**, 375 (2001).
[8] D. Langlois, L. Sorbo, and M. Rodriguez-Martinez, Phys. Rev. Lett. **89**, 171301 (2002).
[9] E. Kiritsis, G. Kofinas, N. Tetradis, T. N. Tomaras, and V. Zarikas, J. High Energy Phys. **0302**, 035 (2003).
[10] T. Tanaka and Y. Himemoto, Phys. Rev. D **67**, 104007 (2003).
[11] T. Shiromizu, K. I. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
[12] A. Chamblin, A. Karch, and A. Nayeri, Phys. Lett. B **509**, 163 (2001).
[13] P. C. Vaidya, Nature (London) **171**, 260 (1953); R. W. Lindquist, R. A. Schwartz, and C. W. Misner, Phys. Rev. **137**, B1360 (1965).
[14] R. R. Caldwell and D. Langlois, Phys. Lett. B **511**, 129 (2001).
[15] K. Ichiki, M. Yahiro, T. Kajino, M. Orito, and G. J. Mathews, Phys. Rev. D **66**, 043521 (2002).
[16] V. Barger, J. P. Kneller, H. S. Lee, D. Marfatia, and G. Steigman, Phys. Lett. B **566**, 8 (2003).
[17] A. Hebecker and J. March-Russell, hep-ph/0103214 V4.